NEW TRENDS IN RINGS AND MODULES I
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TOPOLOGIES ON THE SET OF SECOND SUBMODULES

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Abstract. Let R be a ring with identity and M be a unital right R-module. A nonzero submodule N of M is called a second submodule if N and all its nonzero homomorphic images have the same annihilator in R. Let $Spec^s(M)$ denote the set of all second submodules of an R-module M. Recently this submodule class has been studied in detail in number of papers (for example [**?**],[**?**], [**?**],[**?**]) to find the relationships betwee algebraic ad topological properties. In this talking, we study some properties of second submodule and some interrelations between algebraic properties of a module M and topological properties of the second classical Zariski topology on $Spec^s(M)$. We prove that a right R-module M has only a finite number of maximal second submodules if and only if $Spec^s(M)$ is a finite union of irreducible closed subsets. We observe some interrelations between compactness of the second classical Zariski topology of a module M and finiteness of the set of minimal submodules of M. We prove a connection between connectedness of $Spec^s(M)$ and decomposition of M for a right R-module M. We get several characterizations of a noetherian module M over a ring R such that every right primitive factor of R is artinian for which $Spec^s(M)$ is connected. We also study various properties of $Spec^s(M)$ with respect to different topologies and investigate the dual Zariski topology from the point of view of separation axioms, spectral spaces and combinatorial dimension. We obtain characterizations of commutative Quasi-Frobenius and artinian rings by using topological properties of the second classical Zariski topology.

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References

[1] S. Annin, Attached primes over noncommutative rings, J. Pure Appl. Algebra, 212 (2008), no. 3, 510-521.

[2] H. Ansari-Toroghy and F. Farshadifar, The Zariski topology on the second spectrum of a module, Algebra Colloquium, (2014) 21, 671.

[3] H. Ansari-Toroghy, S. Keyvani and F. Farshadifar, On the second spectrum of a module (II), Bull. Malays. Math. Sci. Soc. (2016) 39;1089-1103.

[4] S. Çeken and M. Alkan, Dual of Zariski topology for modules, Book Series: AIP Conference Proceedings, 1389 (1), (2011), 357-360.

[5] S. Çeken, M. Alkan, On graded second and coprimary modules and graded secondary representations, Bull. Malays. Math. Sci. Soc. (2013).

[6] S. Çeken, M. Alkan, On the second spectrum and the second classical Zariski topology of a module, Journal of Algebra and Its Applications Vol. 14, No. 8 (2015) .

[7] S. Yassemi, The dual notion of prime submodules, Arch. Math (Brno) 37 (2001), 273–278..