

A GENERALIZATION OF EXCHANGE RINGS

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We define and explore a new class of rings called unit-exchange rings that strictly contain the class of exchange rings. We will show that the definition of unit-exchange is left-right symmetric for each element. We will show several classes of rings that are unit-exchange but not exchange. It is known, due to Kaplansky, that a von Neumann regular ring R is unit regular if and only if it has stable range one. Later Camillo and Yu extended this result to exchange rings. We first show that the property of stable range one is equivalent to the property that every left (right) unit lifts modulo every left (right) principal ideal. Using this property, we generalize Camillo and Yu's result and show that for a unit-exchange ring the notion of stable range one is equivalent to the property that the ring is partially unit-regular which is equivalent to the property that the ring is left (right) uniquely generated.

Keywords: exchange rings, suitable rings, von Neumann regular, stable range one, uniquely generated, 2-good, units .

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