

$\oplus - G$ -SUPPLEMENTED MODULES

Celil Nebiyev¹ and Hasan Hüseyin Ökten²

¹*Department of Mathematics, Ondokuz Mayıs University 55270
Kurupelit-Atakum/Samsun/TURKEY*

²*Technical Sciences Vocational School, Amasya University, Amasya/TURKEY*

ABSTRACT. Let M be an R -module. If every submodule of M has a g -supplement that is a direct summand of M , then M is called a $\oplus - G$ -supplemented module. In this work, some properties of these modules are investigated.

Results

Lemma 1.1. *Let M be an R -module, V be a supplement of U in M and $X, Y \leq V$. Then X is a g -supplement of Y in V if and only if X is a g -supplement of $U + Y$ in M .*

Corollary 1.2. *Let $M = M_1 \oplus M_2$ and $X, Y \in M_2$. Then X is a g -supplement of Y in M_2 if and only if X is a g -supplement of $M_1 + Y$ in M .*

Lemma 1.3. *Let M be an R -module and $M = M_1 \oplus M_2$. If M_1 and M_2 are $\oplus - G$ -supplemented, then M is also $\oplus - G$ -supplemented.*

Corollary 1.4. *Let M be an R -module and $M = M_1 \oplus M_2 \oplus \dots \oplus M_n$. If M_i $\oplus - G$ -supplemented for every $i = 1, 2, \dots, n$, then M is also $\oplus - G$ -supplemented.*

Proposition 1.5. *Let M be a $\oplus - G$ -supplemented module. If every g -supplement submodule in M is a direct summand of M , then every direct summand of M is $\oplus - G$ -supplemented.*

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E-mail addresses: ¹cnebiyev@omu.edu.tr , ²hokten@gmail.com .

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