ON A CERTAIN COMMUTATIVITY THEOREM FOR RINGS

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Abstract. All considered rings are associative, not necessarily with the unity. The problem of determining conditions that are sufficient for a ring to be commutative has been studied by numerous mathematicians for many years and by various methods. It seems that most well known results in this subject are theorems of Herstein and Jacobson:

Theorem(Jacobson, Theorem 11 of [7]) Let R be a ring in which for every element a there is an integer $n(a) > 1$, depending on a, $a = a^{n(a)}$. Then R is commutative.

Theorem(Herstein, Theorem 18 of [6]) If R is a ring with center C, and if $x^n - x \in C$ for all $x \in R$, n is a fixed integer larger than 1, then R is commutative.

This talk is devoted to the investigation of the structure of torsion-free rings R satisfying condition:

$$\forall_{x\in R}\exists_{m,n\in\mathbb{Z}\setminus\{0\}}\ \ mx^2 = nx.$$

It turns out that any ring that fulfills the above condition is not only commutative ring but it is a subring of the field $\mathbb{Q}$ of rational numbers. We will present the applications of this fact in the classification of filial rings (i.e., rings in which the relation of being an ideal is transitive).

This is a joint work with R. Andruszkiewicz.

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