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A NOTE ON A GENERALIZATION OF INJECTIVE MODULES

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Abstract. Initially, Zöschinger generalized injective modules to modules with the property (E) in [10]. Motivated by [10], we investigate that how to obtain another proper generalization of injective modules. So we define modules has the property (ME) on the purpose of attaining generalization of injective modules via mutual supplements. We say that a module M has *the property (ME)* if, whenever $M \subseteq N$, M has a supplement K in N where K has a mutual supplement in N. The aim of this study to give some properties of modules has the property (ME). Then we prove that the property (E) and the property (ME) are conflict in the notion of semisimple modules. Finally we show that a principal ideal domain R is left perfect if every free left R-module has the property (ME).

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