

FAITH'S PROBLEM ON R -PROJECTIVITY IS INDEPENDENT
OF ZFC

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ABSTRACT. In [1], Faith asked for a characterization of the rings R such that each R -projective module is projective, that is, the Dual Baer Criterion holds in $\text{Mod-}R$. Such rings were called right testing. Sandomierski [3] proved that each right perfect ring is right testing. Puninski et al. [2] have recently shown for a number of non-right perfect rings that they are not right testing (in ZFC), and noticed that [4] proved consistency with ZFC of the statement 'each right testing ring is right perfect.'

We prove the complementing consistency result: the existence of a right testing, but non-right perfect ring is also consistent with ZFC. Thus the answer to the Faith's question above is independent of ZFC, [5]. Moreover, for each cardinal κ , we provide examples of non-right perfect rings R such that the Dual Baer Criterion holds (in ZFC) for all $\leq \kappa$ -generated R -modules.

References

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- [4] J. TRLIFAJ: *Whitehead test modules*, *Trans. Amer. Math. Soc.* **348** (1996), 1521-1554.
- [5] J. TRLIFAJ: *Faith's problem on R -projectivity is undecidable*, preprint, arXiv:1710.10465v1.