

## PURE-PROJECTIVITY FROM A DIFFERENT PERSPECTIVE

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ABSTRACT. Given modules  $M$  and  $N$ ,  $M$  is said to be  $N$ -pure-subprojective if for every pure epimorphism  $g : B \rightarrow N$  and homomorphism  $f : M \rightarrow N$ , there exists a homomorphism  $h : M \rightarrow B$  such that  $gh = f$ . For a module  $M$ , the pure-subprojectivity domain of  $M$  is defined to be the collection of all modules  $N$  such that  $M$  is  $N$ -pure-subprojective. A module is pure-projective if and only if its pure-subprojectivity domain consists of all modules. As pure-subprojectivity domains clearly include all pure-projective modules, a reasonable opposite to pure-projectivity in this context is obtained by considering modules whose pure-subprojectivity domain consists of only pure-projective. It is thus to keep in line with [3], we refer to these modules as pp-indigent. This study is inspired by similar ideas and problems in [1,2,3]. Properties of pure-subprojectivity domains and of pp-indigent modules are studied. The structure of a ring over which every (simple, finitely generated) right module is pp-indigent or pure-projective is investigated.

This is a joint work with Yılmaz Durğun.

### References

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