

## COFINITELY RAD-SUPPLEMENTED LATTICES

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ABSTRACT. In this work, all lattices are complete modular lattices. Let  $L$  be a lattice. If every cofinite element of  $L$  has a Rad-supplement in  $L$ , then  $L$  is called a cofinitely Rad-supplemented lattice. In this work, some properties of these lattices are investigated.

### Results

**Lemma 1.1.** *Let  $L$  be a lattice,  $a \in L$  and  $x$  be a cofinite element of  $L$ . If  $x \vee a$  has a Rad-supplement in  $L$  and  $a/0$  cofinitely Rad-supplemented, then  $x$  has a Rad-supplement in  $L$ .*

**Lemma 1.2.** *Let  $L$  be a lattice,  $I$  be a nonempty index set and  $a_i \in L$  for every  $i \in I$ . If  $1 = \bigvee_{i \in I} a_i$  and  $a_i/0$  is cofinitely Rad-supplemented for every  $i \in I$ , then  $L$  is also cofinitely Rad-supplemented.*

**Corollary 1.3.** *Let  $L$  be a lattice and  $1 = a_1 \vee a_2 \vee \dots \vee a_n$  in  $L$ . If  $a_i/0$  is cofinitely Rad-supplemented for every  $i = 1, 2, \dots, n$ , then  $L$  is also cofinitely Rad-supplemented.*

**Proposition 1.4.** *Let  $L$  be a cofinitely Rad-supplemented lattice and  $a \in L$ . Then  $1/a$  is also cofinitely Rad-supplemented.*

**Proposition 1.5.** *Let  $L$  be a cofinitely Rad-supplemented lattice. Then every cofinite element of  $1/r(L)$  is a direct summand of  $1/r(L)$ .*

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